Aust. Met. Mag. 50 (2001) : 171-182

THE APPLICATION OF WEATHER DERIVATIVES TO MITIGATE THE FINANCIAL RISK OF CLIMATE VARIABILITY AND EXTREME WEATHER EVENTS.

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(Manuscript received, November 2000, revised January 2001)

Evidence of the challenge faced by the meteorological community to become skilled in applying risk management products from the financial markets is growing. This paper presents an approach to the pricing of weather derivatives that employs a combination of empirical data including forecast verification data, regional synoptic classification data, and data associated with climate indices on a global scale, such as the Southern Oscillation Index. The paper presents several illustrative examples that show how to price these options about the occurrence of an unusual weather event, using forecast verification data and synoptic classification data.

1. INTRODUCTION

Evidence of the challenge faced by the meteorological community to become skilled in applying risk management products from the financial markets is growing (Dischell, 2000).

Papers presented to recent meteorological and environmental applications symposia, books such as *"Insurance and Weather Derivatives"* (Geman, 1999) and articles in prestigious journals, such as *Risk* and *Energy and Power Risk Management* are testimony to the increasing importance of weather derivatives. The advent of weather derivatives raises issues such as quality control of data, ensuring that data are free from corruption, exchange of data, observational site security, and legal liability (Clewlow *et al.,* 2000).

The purpose of the present paper is to present an empirical approach to the pricing of weather derivatives. It shall be seen that there are similarities between the approach used in the present paper and that of Zeng (2000). Both take samples from the historical climate record to develop pricing models. Specifically, Zeng (2000) employed:

- An historical data base of the weather index (or indices) to be subsequently used in the pricing of the weather derivative contract (or contracts);
- A corresponding historical temperature and/or precipitation data base;
- The official seasonal forecast of the expected probability distribution of temperature and/or precipitation;

to yield the expected probability distribution of the weather index (or indices), from which the pricing is derived.

However, the present paper builds upon Zeng's (2000) work by incorporating a combination of additional data types such as:

- Forecast error data;
- Regional synoptic classification data;
- Data associated with climate indices on a global scale, such as the Southern Oscillation Index (SOI).

Stern (1996a) employed empirical stock market data to improve upon the option pricing derived using the Black and Scholes (1973) theoretical

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model. It is, therefore, considered that one might also improve upon current weather derivative prices by using a combination of the aforementioned additional empirical data.

The approach may, in particular, be applied to managing the risk associated with extreme cases of weather parameters, such as temperature and precipitation. The approach is demonstrated with several illustrative examples.

Indeed, Stern (1999) reports a case of extreme (and unjustified) price volatility in agricultural commodities arising from traders' being unaware of what forecast verification data might have suggested.

2. BACKGROUND

The earliest published work on the subject is that by the current author (Stern, 1992), who employed option-pricing theory to establish a measure of the economic consequences of changes in the global mean temperature. These were examined across scales from the macro- to the micro-economy, being replicated by a combination of *Global Mean Temperature* futures contracts and an associated set of option contracts.

It is of interest that Stern's work, which was undertaken at the (Australian) Bureau of Meteorology, was prompted by a 1991 call by the Australian Electricity Supply Industry Research Board to conduct greenhouse research related to electricity supply. It is, indeed, the energy and power industries that have, so far, taken best advantage of the opportunities presented by weather derivatives.

The subject of weather derivatives is first discussed in an energy and power industry journal by Simpson (1996/97). Clemmons *et al...*(1999) report the first weather derivative contract as a "temperature-related power swap ... transacted in August 1996".

By the end of 1997, some 150 weather derivative deals had been completed, whilst by the end of 1998, this figure had increased to 500. An Internet weather derivatives exchange was launched in January 2000. The first Australian deal was transacted in April 2000. Of the 3500 transactions completed by the middle of the year 2000, 98% were based on temperature, of which most were Degree Day contracts constructed for the energy and power industry.

3. WEATHER DERIVATIVES DEFINED

Jain and Foster (2000) observe that "the world is heavily affected by the weather. But while there is little anyone can do to control the climate, businesses can now mitigate the exposure they face from adverse weather conditions by using weather derivatives".

These businesses include:

- Energy and power;
- Agriculture and agrochemicals;
- Viticulture;
- Brewing;
- Clothing;
- Construction;
- Theme parks;
- Retail food and drink;
- Tourism;
- Sporting;
- Outdoor entertainment;
- Water authorities and irrigation.

Clewlow *et al...*(2000) describe a derivative as "a financial product that derives its value from other more basic variables". These products include:

- Futures;
- Forwards;
- Call options;
- Put options;
- Swaps.

They describe weather derivatives as being similar "to conventional financial derivatives, the basic difference coming from the underlying variables that determine the payoffs", such as:

- Temperature;
- Precipitation;
- Wind;
- Heating Degree Days;
- Cooling Degree Days.

Weather derivative contracts are typically defined by:

- Location;
- Type of asset (e.g. Heating Degree Days);
- Strike (value of underlying asset at which one party is obliged to compensate the other);
- Expiry (the time when one party is obliged to compensate the other);
- Notional ($ per unit of underlying asset).

Dischell (1998) notes that "Traditional weather insurance ... requires a demonstration of loss (whereas) weather derivatives ... require no demonstration of loss and provide protection from the uncertainty in normal weather".

This protection is achieved because a weather derivative contract, when applied as a hedge, sets limits on how far revenues can fall and expenses can increase. On the other side of such a contract may be a speculator, to whom the risk has been transferred in return for a reward.

Alternatively, on the other side of the contract may be another hedger, who wishes to protect against loss associated with the opposite scenario (e.g. high temperatures) to the scenario that the counter-party is concerned about (e.g. absence of high temperatures).

So, like all derivatives, weather derivatives may be used to transfer risk from those who are involuntarily exposed to unwanted risk to those who have a traditional familiarity with risk.

4. CALL OPTIONS

Our first illustrative example is that of a *Cooling Degree Day call option*. A call option contract gives the holder the right (but not the obligation) to buy (that is, to "call"), from the seller of the call option, a quantity of a particular commodity on or before a certain date at a certain price.

The commodity may be something "real" (that is, a parcel of shares) or the commodity may be something "esoteric" (for example, a "parcel" of Cooling Degree Days).

Before proceeding, let us refer to a basic text on options in order to develop a more complete understanding of the meaning of the term "call option". Cox and Rubinstein (1985) note that "options markets exist for a wide variety of instruments, so as to avoid needless repetition we will focus on the oldest and largest of these markets, options on common stocks".

They define a call option as:

A contract giving its owner the right to buy:
-a fixed number of shares (1000 shares in Australia)
-of a specified common stock (known as the underlying security)
-at a fixed price (known as the strike price)
-at any time on or before a given date (known as the expiry date).

This definition applies only to what is termed an *American* call option. A *European* call option has identical features to an *American* call option, except that the phrase "at any time on or before a given date" is replaced by "on a given date". Options traded on the Australian Stock Exchange are *American* in style. Cox and Rubinstein (1985) further define:

-the act of making this transaction as exercising the option
-the individual who creates the call as the seller or the writer
-the individual who purchases the call as the holder or buyer
-the market price of the call as the premium or price

They note that if a call is exercised, the complete transaction involves an initial exchange of:

-call price from the buyer to the seller
-call option from the seller to the buyer

and a subsequent exchange of:

-strike price and call option from the buyer to the seller
-common stock from the seller to the buyer

For example, a TELSTRA $7.25 December call option bought at the close of trading on August 11, 2000, when TELSTRA shares were trading at $7.12, would have cost $0.38 a share (*Australian Financial Review*, 14 August 2000). This call gave the buyer the right to purchase 1000 TELSTRA shares for $7.25 per share at any time until the end of December, 2000. On any trading day until the expiry date, the buyer can do one of three things:

-sell the call back at its concurrent price, thereby cancelling the position
-exercise the call by payment of $7250.00 in return for 1000 shares
-retain the call and do nothing

On the expiry date the third alternative is equivalent to permitting the call to expire.

Suppose that, on the expiry date, TELSTRA shares were $8.00. The buyer has the right to purchase 1000 shares for $7.25 each. The buyer may exercise that right. Alternatively, with the likely price of the option at $0.75, the buyer may choose to sell it. In this circumstance, however, the buyer would not choose to "do nothing".

Alternatively, suppose that, on the expiry date, TELSTRA shares were $6.00. The buyer has the right to purchase 1000 shares for \$7.25 each. In this circumstance, however, the buyer would obviously not choose to exercise the option. Furthermore, the value of the option would be zero, so the buyer would not have the opportunity to sell it. In this circumstance, the buyer would "do nothing". With this background explanation of what call options are, we now proceed to a weather-related application, that of a Degree Day call option.

The number of Cooling Degree Days during a season (which might be regarded as a measure of the requirement for cooling) is the accumulated number of Degrees that the daily mean temperature is above a particular base figure, usually 18°C. If the average temperature on a particular day is below 18°C, there is no contribution for that day. Heating Degree Days (which might be regarded as a measure of the requirement for heating) are defined in the reverse way. Electricity retailers are concerned with periods of extreme heat and cold, on account of the resulting heavy consumer demand leading to "spikes" in prices, and by purchasing Degree Day call options, they reduce their risk.

Let us now suppose that an electricity retailer holds a Cooling Degree Day call option. That option may be viewed as placing a maximum limit on the number of Degree Days during a season (the "strike") before the electricity company is entitled to purchase a "parcel" of Cooling Degree Days at a pre-determined price, regardless of the price of the electricity. What this in effect means is that if, at the expiry of the contract, the actual number of Cooling Degree Days is greater than the strike, the seller of the option pays the buyer a certain amount.

Firstly, let us define our weather derivative contract thus:

- Location: Not specified in this case;
- Type of asset: Cooling Degree Days;
- Strike: 600 Cooling Degree Days;
- Expiry: Not specified in this case;
- Notional: \$100 per Cooling Degree Day above 600.

If, at the expiry of this contract, the accumulated number of Cooling Degree Days is greater than the strike (600), then the seller of the option pays the buyer the notional (\$100) for each Cooling Degree Day above the strike. This is illustrated in the pay-off diagram at Figure 1.

For example, suppose that the accumulated Cooling Degree Days at the expiry is 1400. The "pay-off" is then \$(1400-600)x100=\$80000.

5. PRICING APPROACHES

There are three different approaches to the pricing of weather derivatives. These are:

- Historical simulation - *this involves computing the historical pay-off of a derivative via statistical analysis of past data*;
- Direct modelling of the underlying variable's distribution - *this involves modelling the underlying as a normally, or as a log-normally, distributed variable (similar to short-term forecasting);*
- Indirect Monte Carlo modelling of the underlying variable's distribution - *this involves simulating a sequence of data, allowing for the incorporation of accurate seasonal patterns, mean reversion, jumps and changing volatility.*

Historical simulation originated in the insurance industry. The approach asks: "What would be the pay-out, on average, had the company sold the option every year for the last *n* years? For example, given the last 30 years of temperature data, we can calculate 30 samples of the pay-off for a Cooling Degree Day option for the month of January in Melbourne. The approach is useful because it allows development of an indicative pricing methodology.

In order to establish what the appropriate distribution is, we look at the underlying variable's mean and standard deviation over the relevant period. In some cases the distribution may be approximately lognormal; in other cases it appears more normal. A model, that is frequently applied to financial market variables is one developed by Black and Scholes (1973). Cox and Rubinstein (1985) write the Black and Scholes call option pricing formula:

$$C=SN(x)-Kr^{-t}N(x-\sigma\sqrt{t}) \text{ where } x = \frac{\ln(S/Kr^{-t})}{\sigma\sqrt{t}} + (1/2)\sigma\sqrt{t}$$

where:

C=call option price
r=1+interest rate
K=option strike price
S=current market price
t=time to expiry of option
N=normal density function
σ=standard deviation of security's return

and the put option pricing formula:

$$P = Kr^{-t}N(y+\sigma\sqrt{t}) - SN(y) \text{ where } y = \frac{\ln(Kr^{-t}/S)}{\sigma\sqrt{t}} - (1/2)\sigma\sqrt{t}$$

and P=put option price

The key assumption of the Black and Scholes model is that the variable underlying the option is lognormally distributed. The model (therefore) suggests that the underlying variable can increase without limit. Weather variables, such as temperature, tend to remain within relatively narrow bands - a "mean-reverting" type of behaviour. As a result, it is considered that the Black and Scholes model has deficiencies when applied to weather variables.

Monte Carlo simulation involves simulating a sequence of data. It provides a general and flexible way to price many different weather derivative structures, allowing the use of models that incorporate:

- Seasonal patterns;
- Forecasts;
- Mean-reversion behaviour;
- Extreme events;
- Jumps;
- Changing volatility.

A simple example of Monte Carlo modelling (after Dischell, 1999) is now presented. Where:

a, ***b*** and ***c*** are constants;
T(n+1)=projected temperature for day **n+1**;
T(n)=previously projected temperature for day **n**;
M(n+1)=mean temperature for projected day **n+1**;
Ch(n,n+1)=random change from day **n** to day **n+1**;

the following first-order recurrence relationship is employed to describe the evolution of a temperature sequence:

$$T(n+1) = aT(n) + bM(n+1) + cCh(n,n+1)$$

6. EVALUATING A 38°C CALL

Our second illustrative example is that of a *38°C call option*. This example applies to the case when a temperature of at least 38°C has been forecast. Firstly, let us define our weather derivative contract thus:

- Location: Melbourne
- Type of asset: Temperature (°C)
- Strike: 38°C
- Expiry: Tomorrow
- Notional: $100 per degree above 38°C

If, at the expiry of a call option contract (that is, tomorrow), the actual maximum temperature is greater than the strike (that is, 38°C), the seller of the option pays the buyer $100 for each 1°C it is above 38°C. This is illustrated in the pay-off diagram at Figure 2.

We now determine the price of our call option contract by employing historical simulation of the outcomes. We note that between 1960 and 2000, there were 114 forecasts of at least 38°C. The distribution of historical outcomes is presented in the graphic at Figure 3. From the data presented in Figure 3, it may be seen that the contribution from the historical outcomes to the price of the 38°C call option contract are:

- 1x44°C yields $(44-38)x1x100=$600
- 2x43°C yields $(43-38)x2x100=$1000
- 6x42°C yields $(42-38)x6x100=$2400
- 13x41°C yields $(41-38)x13x100=$3900
- 15x40°C yields $(40-38)x15x100=$3000
- 16x39°C yields $(39-38)x16x100=$1600
- The other 61 cases (38°C or below) yield nothing

...leading to a total contribution of $12500, and an average contribution over the 114 cases of **$110**

So, **$110** is the "fair value" price of our call option. In order to make a profit, the seller would need to negotiate a price in excess of the "fair value" $110.

7. EVALUATING A DAILY RAINFALL RANGE 3 CALL

Our third illustrative example is that of a *Daily Rainfall Range 3 call option*. This example applies to the case when a rainfall range of at least range 3 has been forecast. Firstly, let us define our weather derivative contract thus:

- Location: Melbourne
- Type of asset: Rainfall (range)
- Strike: Range 3
- Expiry: Tomorrow
- Notional: $100 per rainfall range above range 3

where:

- rainfall range 0 = nil
- rainfall range 1 = 0.1 mm to 2.5 mm
- rainfall range 2 = 2.6 mm to 5 mm
- rainfall range 3 = 5.1 mm to 10 mm
- rainfall range 4 = 10.1 mm to 20 mm
- rainfall range 5 = 20.1 mm to 40 mm
- rainfall range 6 = 40.1 mm to 80 mm
- rainfall range 7 = more than 80 mm

If, at the expiry of a call option contract (that is, tomorrow), the actual rainfall range is greater than the strike (that is, range 3), the seller of the option pays the buyer $100 for each rainfall range it is above range 3. This is illustrated in the pay-off diagram at Figure 4.

We now determine the price of our call option contract by employing historical simulation of the outcomes (Figure 5).

We note that between 1982 and 1999, there were 244 forecasts of at least range 3. The distribution of historical outcomes is presented in the graphic at Figure 5. From the data presented in Figure 5, it may be seen that the contribution from the historical outcomes to the price of the Rainfall Range 3 call option contract are

- 3xrange 6 yields $(6-3)x3x100=$900
- 20xrange 5 yields $(5-3)x20x100=$4000
- 52xrange 4 yields $(4-3)x52x100 = $5200
- The other 169 cases (range 3 or below) yield nothing

...leading to a total contribution of $10100, and an average contribution over the 244 cases of **$41**
So, **$41** is the price of our call option.

8. PUT OPTIONS

In order to explain the operation of put options, our fourth illustrative example is that of a *Cooling Degree Day put option*. A put option contract gives the holder the right to sell (that is, to "put") from the seller of the put option, a quantity of a particular commodity on or before a certain date at a certain price.

Before proceeding, let us refer to our basic text on options " (Cox and Rubinstein, 1985) in order to develop a more complete understanding of the meaning of the term "put option". They define a put option as:

> A contract giving its owner the right to sell:
> -a fixed number of shares (1000 shares in Australia)
> -of a specified common stock (known as the underlying security)
> -at a fixed price (known as the strike price)
> -at any time on or before a given date (known as the expiry date).

They note that if a put is exercised, the complete transaction involves an initial exchange of:

> -put price from the buyer to the seller
> -put option from the seller to the buyer

and a subsequent exchange of:

> -common stock and put option from the buyer to the seller
> -strike price from the seller to the buyer

For example, a TELSTRA $7.25 December put option bought at the close of trading on August 11, 2000, when TELSTRA shares were trading at $7.12, would have cost $0.49 a share (*Australian Financial Review*, 14 August 2000). This call gave the buyer the right to sell 1000 TELSTRA shares for $7.25 per share at any time until the end of December, 2000. On any trading day until the expiry date, the buyer can do one of three things:

> -sell the put back at its concurrent price, thereby cancelling the position
> -exercise the put by selling 1000 shares in return for $7250.00
> -retain the put and do nothing

On the expiry date the third alternative is equivalent to permitting the put to expire.

Suppose that, on the expiry date, TELSTRA shares were $8.00. The buyer, or holder, of the option has the right to sell 1000 shares for $7.25 each. The holder may exercise that right. In this circumstance, however, the holder would obviously not choose to exercise the option. Furthermore, the value of the option would be zero, so the holder would not have the opportunity to sell it. In this circumstance, the holder would "do nothing".

Alternatively, suppose that, on the expiry date, TELSTRA shares were $6.00. The holder has the right to sell 1000 shares for $7.25 each. The holder may exercise that right. Alternatively, with the likely price of the option at $1.25, the holder may choose to sell it. In this circumstance, however, the holder would not choose to "do nothing".

Let us now suppose that an electricity retailer holds such an option. That option places a minimum limit on the number of degree days during a season (the "strike") before the electricity company is entitled to sell a "parcel" of cooling degree days at a pre-determined price, regardless of the price of the electricity. What this in effect means is that if, at the expiry of the contract, the actual number of Cooling Degree Days is less than the strike price, the seller of the option pays the buyer a certain amount. This could compensate the retailer for weak sales on account of a cooler than normal summer, which suppresses demand.

Firstly, let us define our weather derivative contract thus:

- Location: Not specified in this case;
- Type of asset: Cooling Degree Days;
- Strike: 600 Cooling Degree Days;
- Expiry: Not specified in this case;
- Notional: $100 per Cooling Degree Day below 600.

This is illustrated in the pay-off diagram at Figure 6. In the case illustrated, suppose, for example, that the accumulated Cooling Degree Days at the expiry of the option equals 300. The "pay-off" is then $(600-300)x100=$30000.

9. EVALUATING AN ECHUCA MONTHLY RAINFALL DECILE 4 PUT

Our fifth illustrative example is that of a *Monthly Rainfall Decile 4 put option*. This example applies to the case when the preceding month's Southern Oscillation Index (SOI) is Decile 1, 2 or 3, i.e. -4.7 or less. Firstly, let us define our weather derivative contract thus:

- Location: Echuca (some 200 km north of Melbourne)
- Type of asset: October Rainfall (Decile)
- Strike: Decile 4
- Expiry: October
- Notional: $100 per Decile below Decile 4

If, at the expiration of a put option contract (that is, October), the actual Rainfall is less than the strike price (that is, Decile 4), the seller of the option pays the buyer $100 for each Decile that it is below Decile 4. This is illustrated in the pay-off diagram at Figure 7.

We now determine the price of our put option contract by employing historical simulation of the outcomes. We note that, between 1876 and 1999, there were 119 Octobers with rainfall records, of which 44 were preceded by months with an SOI of Decile 1, 2 or 3. The distribution of historical outcomes is presented in the graphic at Figure 8.

From the data presented in Figure 8, it may be seen that the contributions from the historical outcomes to the price of the Decile 4 put option contract are

- 9xDecile 1 yields $(4-1)x9x100=$2700
- 6xDecile 2 yields $(4-2)x6x100=$1200
- 4xDecile 3 yields $(4-3)x4x100=$400
- The other 25 cases (Decile 4 or above) contribute nothing

...leading to a total contribution of $4300, and an average contribution over the 44 cases of **$98**

So, **$98** is the price of our put option.

10. EVALUATING A FORECAST TEMPERATURE ERROR PUT

Our sixth illustrative example is that of a *Forecast Temperature Error put option*. This example applies to the case when a temperature of at least 38°C has been forecast. Firstly, let us define our weather derivative contract thus:

- Location: Melbourne
- Type of asset: Forecast error (°C)

  - Error is defined as (forecast temperature minus observed temperature)
- Strike: 0°C
- Expiry: Tomorrow
- Notional: $100 per degree below 0°C

If, at the expiration of a put option contract (that is, tomorrow), the actual forecast temperature error is less than the strike price (that is, 0°C), the seller of the option pays the buyer $100 for each 1°C it is below 0°C. Another way of describing this is to say that the buyer is paid $100 for each 1°C that the forecast under-estimates the maximum temperature.

This allows an electricity retailer, for example, to firstly enter into an agreement to sell electricity on the next day, at a price determined by the expected high temperature (for example, over 38°C). If the retailer then purchases the put option described above, protection is gained should the temperature the next day be higher than forecast leading to even higher electricity prices.

The historical simulation technique illustrated earlier is then applied again to our 40-year data base of forecast and observed temperatures. This yields a suggested price of **$67** for our put option.

We may ask:

- Is the error in the forecast for tomorrow's temperature related to the error in the forecast that was issued for today?
- Should this be taken into account when determining the price of the put option?

Interestingly, the answer is "Yes" to both questions:

- Analysis of the data shows that, when very high temperatures are forecast, the sign of the error in tomorrow's forecast is more likely to be of the same sign as the error in today's forecast (than by "chance").
- This leads to a suggested put option price of **$41**, if today's temperature has been over-estimated, and a suggested put option price of **$75**, if today's temperature

has been under-estimated. The reason why the put option price should be greater if today's temperature was under-estimated is illustrated in Figure 9, which shows that, in such circumstances, the chances of tomorrow's temperature also being under-estimated are greater (40% as against 23%).

We have previously shown that when the historical simulation technique is applied to our data base of forecast and observed temperatures, this yields a (preliminary) suggested price of **$67**, for our put option. However, we have shown that the error in the forecast that was issued for today should be taken into account when determining the price. We may now ask:

- Is the error in the forecast for tomorrow's temperature related to today's synoptic weather pattern?
- Should this also be taken into account when determining the price of the put option?

Interestingly, the answer is (once again) "Yes" to both questions. To illustrate:

- Analysis of the data shows that, when very high temperatures are forecast, and today's synoptic weather pattern is *moderate anticyclonic and NNE* (Treloar and Stern, 1993; Stern, 1999a) tomorrow's forecast is more likely to be an under-estimate than if today's synoptic weather pattern had been *strong anticyclonic and NNE*.
- This leads to a suggested put option price of **$47**, if today's synoptic weather pattern is *strong anticyclonic and NNE*, and a suggested put option price of **$77**, if today's synoptic weather pattern is *moderate anticyclonic and NNE*.

It may be seen, therefore, that both the preceding error and the preceding synoptic pattern have an impact upon the "fair value" of the option. More sophisticated analyses could enable the relative effects of both to be compared and combined.

Interestingly, Stern (1996b) also demonstrated that relationships exist between tomorrow's forecast error, today's forecast error and synoptic pattern.

11. PRICING OTHER DERIVATIVES

Sutton's (1951) statement on the resemblance between meteorology and economics, which was quoted in Stern's (1992) paper, is worthy of repetition here:

"Both deal fundamentally with the problem of energy transformations and distribution - in economics, the transformation of labour into goods and their subsequent exchange and distribution; in meteorology, transformation and distribution of the energy received from the sun. Both systems are subject to extremely capricious external influences".

One should, therefore, not be surprised if the modelling and pricing of weather derivatives eventually leads to improved techniques for the pricing of other derivatives. Indeed, the following analysis reports on progress towards that end.

We have previously observed that a model, that is frequently applied to the modelling of financial market variables and their derivatives, is one developed by Black and Scholes (1973).

We have also observed that the Black and Scholes model has deficiencies when applied to weather variables. For this reason, it has been preferred to employ historical simulation and/or Monte Carlo techniques when modelling weather variables (and this paper's focus has been largely on historical simulation).

Notwithstanding the general use of Black and Scholes in the modelling of financial market derivatives, some aspects of the Monte Carlo approach to the modelling of weather derivatives (e.g. mean reversion and jumps) may be appropriate to financial derivatives. To test this proposition, the following experiment was designed:

- Australian stock price data (for the 25 leading stocks) were extracted for an 18 month period (March 1999 to August 2000), sequences of at least 5 consecutive falls (or rises) being noted (there were 249 such sequences during that period).
- Dates when the sequences reversed were also noted, and it was assumed that on the reversal date an investor bought in the wake of a fall sequence, and short-sold in the wake of a rise sequence.
- It was assumed that the investor closed positions each time an opposite sequence was reversed.

The outcomes of the experiment support the proposition that mean reversion and jumps need to be incorporated into the modelling of stock price derivatives.

Firstly, to illustrate the operation of mean reversion, note that:

- The average (mean) return on all 249 sequences was +4.51% with a standard deviation of 12.15%;
- This mean is different from zero at the 0.1% level of significance, the above zero return reflecting the operation of mean reversion (if it wasn't operating, the average return would be zero);
- This would be particularly important in the pricing of *American style options* where, because of the flexibility about when positions may be opened and closed, the operation of mean reversion could dramatically reduce (or increase) the value of the option over that which would be suggested by Black and Scholes;
- This would be also important in the pricing of *European style options*, although to a lesser degree, because of the lack of flexibility about when positions may be closed.

Secondly, to now illustrate the operation of jumps, refer to Figure 10, which presents the ratio:

(frequency of returns from experiment)
(frequency of returns if distribution normal)

that ratio being presented in half Standard Deviation steps from the mean. Observe that:

- There was a much higher frequency of "extreme" returns than one would have expected had the distribution of returns been normally distributed;
- Specifically, the high frequency of strongly negative returns (3.24 times what one would have expected) reflects cases where "news" has initiated a sudden re-rating of a stock, so that a trend continues well beyond those trends associated with the typical short-term variations in stock prices;
- Conversely, the high frequency of strongly positive returns (1.94 times what one would have expected) reflects cases where stock prices have "over-shot" on "news", requiring a corrective trend that continues well beyond those trends associated with the typical short-term variations in stock prices;
- This much higher frequency of "extreme" returns reflects the operation of jumps - if jumps weren't present, and the distribution was normal, each of the ratios would have been 1.0;
- This would be important in the pricing of options, particularly *American style options*, where the operation of jumps could dramatically increase the value of the option over that which would be suggested by Black and Scholes;
- The frequency of returns is unexpectedly high for the cases close to the average return, and unexpectedly low for the cases moderately distant from the average return - this occurs because the Standard Deviation has been inflated by the extreme cases. Without the extreme cases, the most of the ratios should be closer to 1.0 - Figure 11 shows that they then, indeed, would be.

In conclusion, it is proposed that, in order to effectively take into account the processes of mean reversion and jumps when pricing options, that:

- The frequency distribution of a range of stock price evolutions be developed using historical analogues to the recent stock price sequence;
- For *American style options*, their price be determined on the basis of the "best" of a *range* of closing strategies - current pricing practice results in *only two* closing strategies being considered, that of closing at option expiry and that of closing at dividend payment time.

This proposal to use the analogue retrieval approach is another illustration of the link between meteorology and economics.

Indeed, the current author's pioneering work (Stern, 1980; Stern, 1985) on the development of automated weather forecasting guidance using analogue retrieval techniques led to the first operational system employing the analogue statistics approach.

The reader is invited to see, for example, Dahni and Stern (1995), for a more recent report on that system's ongoing development.

12. CONCLUDING REMARKS

A unique approach to the pricing of weather derivatives has been presented. The approach utilises a combination of empirical data to price weather derivatives and has been illustrated by a range of examples.

The importance of forecast verification data and synoptic classification data in the process has been demonstrated. This suggests that pricing theory may be employed to provide a measure of the value of a forecast prior to the event.

Finally, it has been shown that *mean reversion* and *jumps*, features of the temperature sequence so successfully applied in the Monte Carlo approach to the modelling of weather derivatives, should also be included in the modelling of other derivatives.

13. ACKNOWLEDGEMENTS

The author takes great pleasure in thanking his colleagues at the Bureau's Victorian Regional Office, in particular, Mark Williams, Glen Prank and Patrizia Pace, for their advice during the course of this paper's preparation, and the Associate Editor, Terry Hart, and reviewers Grant Beard and Richard Whitaker, for their helpful suggestions.

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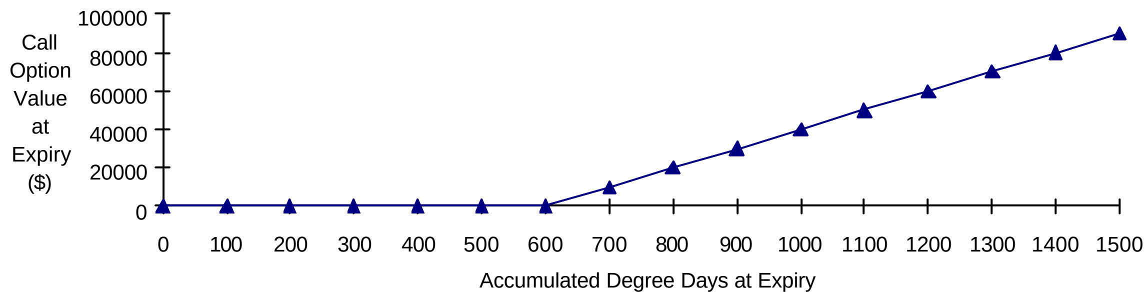

FIGURE 1 Pay-off chart for a Degree Day call.

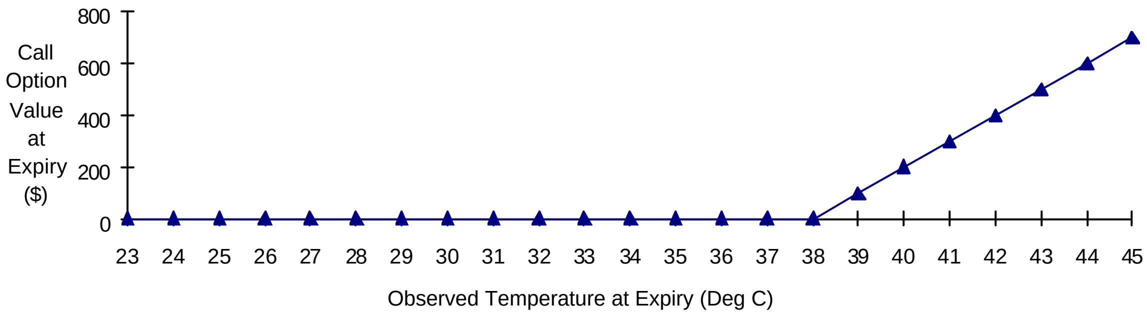

FIGURE 2 Pay-off chart for a 38° call.

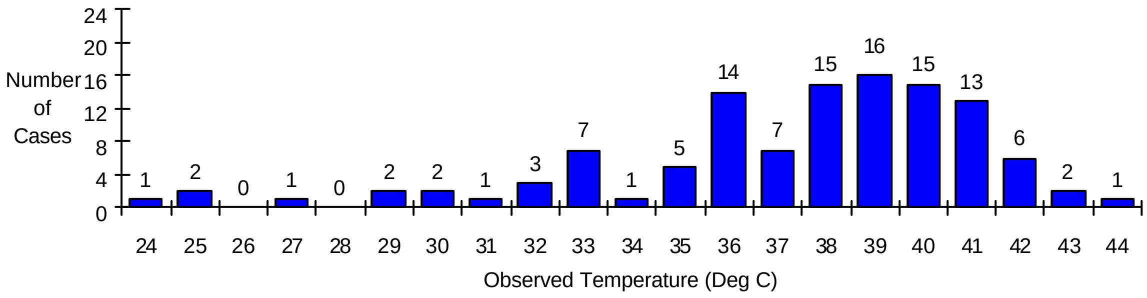

FIGURE 3 Historical outcomes for a 38°C call.

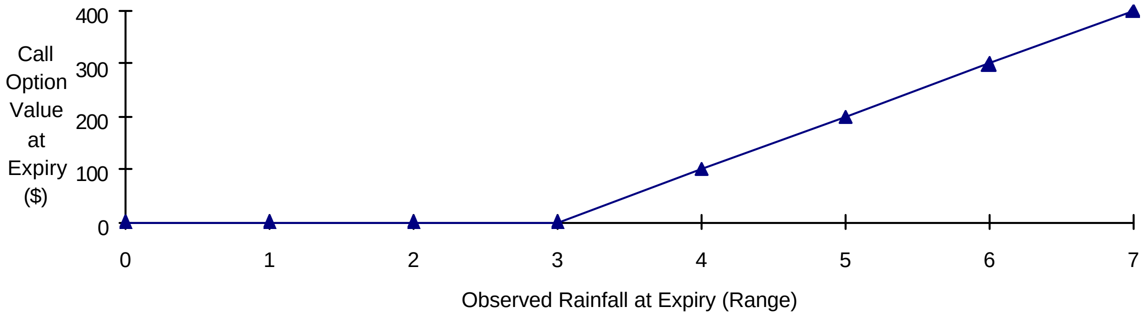

FIGURE 4 Pay-off chart for a Rainfall Range 3 call.

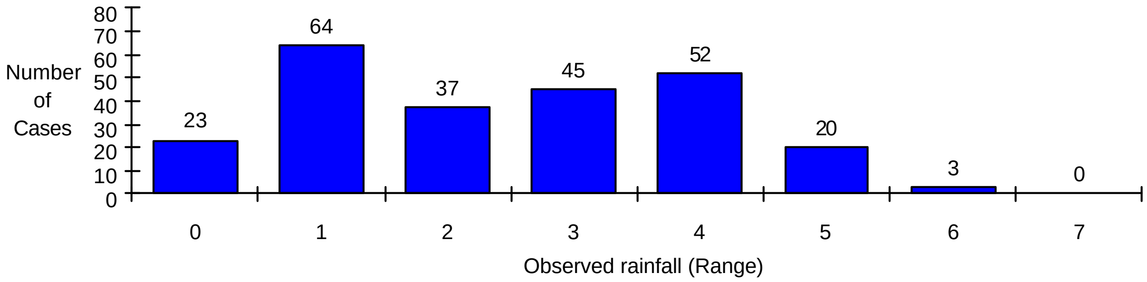

FIGURE 5 Historical outcomes for a Rainfall Range 3 call.

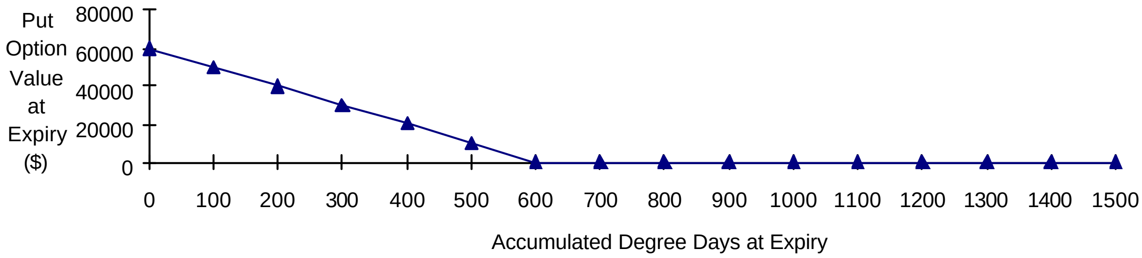

FIGURE 6 Pay-off chart for a Degree Day put.

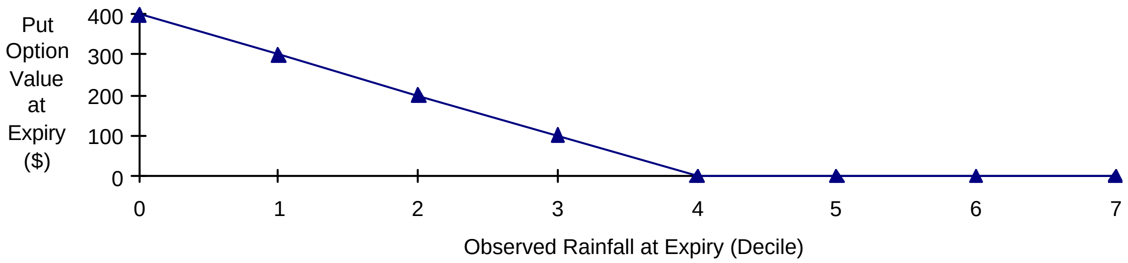

FIGURE 7 Pay-off chart for a Decile 4 put.

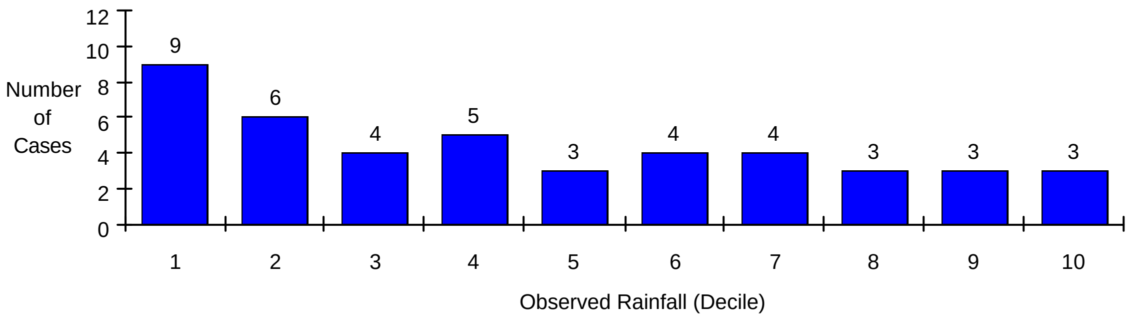

FIGURE 8 Historical outcomes for a Decile 4 put.

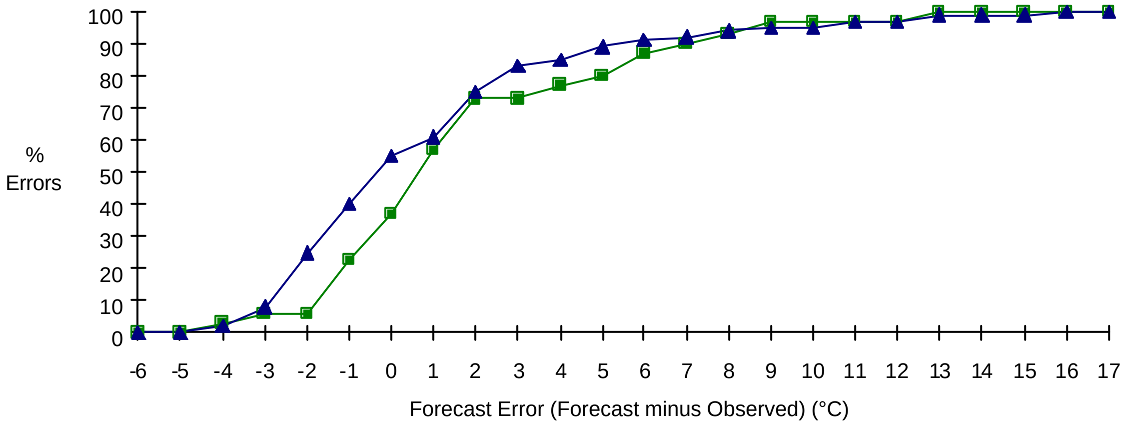

FIGURE 9 Percentage (%) of errors no greater than the value shown for:
(1) Cases when the sign of the error the previous day was +ve (squares); and,
(2) Cases when the sign of the error the previous day was -ve (diamonds).

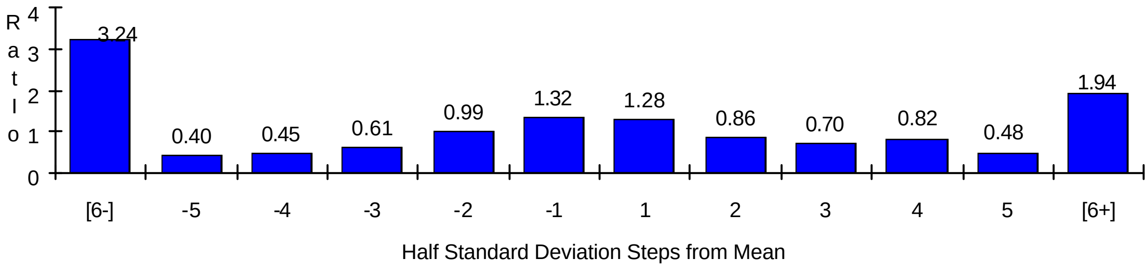

FIGURE 10 The ratio:
(frequency of returns from experiment)/
(frequency of returns if distribution was normal)

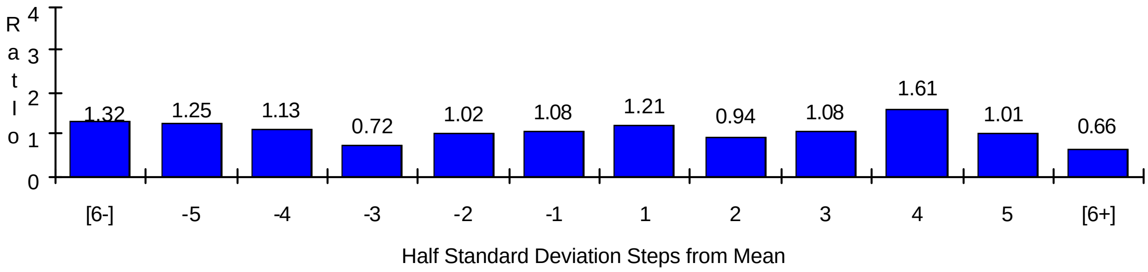

FIGURE 11 The ratio (without extreme cases).