P3.9 ENVELOPING PREDICTOR PARAMETERS IN A REGRESSION EQUATION
- AN ALTERNATIVE TO MODEL OUTPUT STATISTICS (MOS)

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1. INTRODUCTION

This paper describes the application of "parameter enveloping" to weather forecast guidance. Many meteorological services, including the Australian Bureau, have implemented guidance systems based on model output statistics (MOS). MOS involves the derivation of regression equations with predictors including variables output by a numerical weather prediction model (NWP). A weakness of MOS is that frequent changes to NWPs necessitate the re-derivation of the prediction equations on small samples of new data. Also, the user is "locked in" to guidance based on the output of a particular NWP. These weaknesses may be ameliorated by the use of parameter enveloping, employing the "perfect prog" (PP) approach. PP prediction equations are developed on large samples of historical data and the user has the freedom to adjust NWP output if that output appears flawed.

Forecast guidance systems are constrained if they don't account for non-linear effects. Guidance systems often use a set of linear prediction equations, derived by regression techniques, to interpret the output of NWPs in terms of local weather. In Australia, non-linear effects have been partially addressed by the development of a forecast guidance system that retrieves synoptic analogues to initial data or an NWP. To arrive at a prediction, the system statistically interprets either the NWP output or the initial data in terms of future local weather via regression relationships.

This Australian system represents the first comprehensive, operational PP system of weather forecast guidance that is based on monthly and synoptic stratification via analogue retrieval. Initially a pilot model (Stern, 1980 a & b; Stern and Dahni, 1981; Stern and Dahni, 1982), the weather information associated with synoptic analogues from a monthly stratified data set is statistically analysed. It employs a forecast terminology documented in detail by Stern (1980a), to then provide the forecast guidance. Further development of the system (Dahni et al, 1984; Stern et al, 1985, Dahni, 1988) involved its adoption and modification to operational requirements. That the developments largely took place at the Victorian Regional Office of the Australian Bureau of Meteorology endorses Clarke's (1978) philosophy that an "Australian counterpart (to MOS)...will perhaps best be done at the regional level to meet regional problems".

2. PARAMETER ENVELOPING

The process of parameter enveloping, in its simplest form, may be illustrated as follows:

(i) suppose you need to derive a regression equation to forecast a value for predictand $F$ using predictors $P$ and $Q$;

(ii) linear regression, if applied to the data, leads to an equation of the form

$$F = a + bP + cQ,$$

where $a$, $b$, and $c$ being constants;

(iii) in order to establish the spectrum of prediction equations, that might describe the variation in the value of the constants with respect to the westerly and northerly components of the surface flow, $u$ and $v$, a new regression equation is derived that includes as predictors all the terms in the product $(1+u+v)^{(1+P+Q)}$. This leads to a prediction equation of the form

$$F = a + bP + cQ + du + ev + uP + guQ + hvP + ivQ,$$

so providing "envelopes" which define how the surface flow affects the manner in which the other predictors affect the value of the predictand. For example, the partial derivative of $F$ with respect to $P$ is $b + du + hv$. 

Stratification of a data set via synoptic typing (Stern et al, 1985), analogue retrieval,
or month, reduces its size. Some instability in derived regression relationships may result because of the reduced data base. This instability is overcome by the use of parameter enveloping on the entire data set. This leads to a single equation that describes the physical relationship between the predictors and predictand and which, when solved, provides a forecast.

Parameter enveloping may also be seen as "analogue statistics" (AS) in another guise and be regarded as operating in a manner equivalent to taking into account information from neighbouring (unselected) analogues.

3. DISCUSSION

The process of parameter enveloping is now illustrated by an example. The example is a prediction equation for the forecasting of maximum temperature at Melbourne. The predictors, which reflect many of the features of AS, form 3 sets, which are "enveloped" about each other. The sets are:

**Set One** (seasonal):
- sine day of year, cosine day of year (sn, cn)

**Set Two** (MSL pressure gradients):
- 4 predictors, functions of difference in MSL pressure between Melbourne and 350 km to the north (n), east (e), south (s) and west (w).

**Set Three** (miscellaneous):
- 850 hPa temperature (tt)
- Yesterday's maximum over the inland (mi)
- Yesterday's maximum over the coast (mc)
- northerly component of 500hPa wind (v)
- westerly component of 500hPa wind (u)
- a predictor, which is a function of difference in MSL pressure between Melbourne and a location 2000 km to the west (ww)

Inclusion of all combinations of the terms in each set with the terms in the other sets leads to an equation with 104 terms. The equation derived by this means allows an analyst to diagnose the extent to which the importance of physical processes associated with a particular weather element varies as a function of other processes.

The 10 most significant terms in the maximum temperature prediction equation, ranked in descending order of significance as measured by the 't' test are:

\[
\begin{align*}
\text{tt} & : t=9.49 & \text{cn}^{*}tt & : t=8.25 \\
\text{ww} & : t=-8.03 & \text{cn}^{*}ww & : t=-7.06 \\
\text{cn}^{*}tt & : t=-6.95 & \text{mc} & : t=6.62
\end{align*}
\]

To illustrate the equation's potential to diagnose, terms \(tt\) and \(cn^{*}tt\) may be examined in combination. This pair of terms underlines the importance of \(tt\) and indicates its increased importance during the summer when the mixing depth would be greater due to greater insolation.

4. EVALUATION

The equation is derived for Melbourne using the same data (June 1957 - May 1979) as used for Jasper and Stern's (1983) synoptic classification. An evaluation of the performance of the equation on the developmental data reveals a Root Mean Square Error (RMSE) of 1.88 deg C.

In order to assess the stability of the equation, it is solved for an independent data sample covering the 5-year period Jan 1980 to Dec 1984 inclusive. The resulting RMSE of 1.90 deg C suggests that the equation is most stable. Application of a two-tail test to the two RMSEs, at both the 5% and 10% levels of significance, indicates rejection of the hypothesis that the RMSEs are different.

The value of the equation for predicting Melbourne's maximum temperature, is now assessed. By applying the equation to data from the output of the NWP, the assessment is conducted for available data across a two-year period from Jan 1985 to Dec 1986. This period is selected as these years are also used by Dahni (1988) to evaluate the analogue statistics (AS) model. The assessment results in an RMSE of 2.69 deg C for the equation. The RMSE for the corresponding official (issued to the media) forecasts is 2.50 deg C.

Application of a two-tail test suggests rejection of the hypothesis that the official forecasts' RMSE is different from the RMSE achieved by the equation at the 5% level of significance but only marginally so - acceptance is suggested at the 10% level.

With the AS RMSE at 2.59 deg C (Dahni,1988) the two-tail test suggests rejection, at both the 5% and 10% levels, of the hypothesis that its performance is different from that of either the official forecasts or the equation. This result is to be expected as forecasters had the benefit of the AS guidance. It is encouraging as "parameter enveloping" is intended to replicate AS's maximum temperature prediction process.
Examination of a number of situations where the equation and official forecasts are quite different suggests that forecasters disregard the output of the numerical model when they consider it to be flawed. There is, therefore, reason to expect the official forecasts to be more accurate than a strictly objective method, such as the equation, that relies on the numerical output alone.

A one-tail test is, therefore, applied to the two RMSEs and this suggests acceptance of the hypothesis that the official forecasts' RMSE is, at the 5% level of significance, different from and smaller than the RMSE for forecasts derived using the equation.

The reason for this difference in RMSEs is now explored. The official forecasts are not necessarily derived directly from the NWP. On occasions, the numerical prognosis is considered, by forecasters, to be flawed. In such circumstances, the official forecasts may be based on an alternative prognosis. The difference appears to arise because of the forecasters' skill at discriminating between good and flawed NWP.

One might expect an objective method of interpreting numerical prognoses to outperform a subjective method when the prognoses are acceptable. Hence, a one-tail test is applied to the RMSEs achieved by the equation (2.13 deg C) and the official forecasts (2.25 deg C) on the subset of situations designated as not being associated with flawed prognoses ("flawed" being defined as where the equation using NWP data yields an outcome greater than 4 deg C different to the equation using observed data). This test suggests rejection of the hypothesis, at the 5% level of significance, that the equation-based predictions display superior skill to the official predictions, but only marginally so - acceptance is suggested at the 10% level.

Application of a one-tail test to the RMSEs achieved by the equation using prognostic data (5.95 deg C) and the official forecasts (4.27 deg C) on the subset of situations designated as being associated with flawed prognoses, suggests acceptance of the hypothesis, at the 5% level of significance, that the official predictions display superior skill to the equation-based predictions.

The tests verify the forecasters' ability to identify, in advance, flawed NWP and that the equation is at least as effective as the official forecasts when the numerical prognosis is not flawed.

5. CONCLUDING REMARKS

Parameter enveloping facilitates the derivation of a single equation, via regression, to describe the physical relationship between predictors and a predictand. The equation is at least as effective as forecasters at interpreting NWP. As forecasters are effective judges of the likelihood of these NWP being accurate, the PP approach described here, parameter enveloping, is preferred to MOS. The success of parameter enveloping in the context of forecasting Melbourne's maximum temperature, has prompted its extension to other locations and to the prediction of rainfall.

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7. REFERENCES